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Scaling, Light-Cone Expansion, and the Van Hove Model*

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With certain assumptions on the coupling of two currents to particles of increasing spin, it is shown that the Van Hove model results in Bjorken scaling and Regge asymptotic behavior. The fields corresponding to these particles are related to the products appearing in the operator-product expansion near the light cone.

The Bjorken scaling limit[1] for deep-inelastic electron scattering, or more generally for the scattering of any current in the appropriate kinematic region, may be accounted for by the behavior of products of currents close to each other's light cone.[2–5] This scaling limit can be made consistent with Regge asymptotic[6] behavior; such a behavior may be suggested by the data on inelastic electron scattering.[7] In this note we shall point out how these results may be achieved in the context of the Van Hove model.[8] It may likewise shed some light on the nature of the bilocal operators appearing on the right side of the operator-product expansions.[3] It should be emphasized that none of the results will be derived; they will all be inserted into the model from the start. Our purpose is to show the consistency of these assumptions within a dynamical scheme, and as mentioned previously, to discuss their connection with the operator-product expansion.

For brevity we shall consider the scattering of a current by a spinless particle and study only the even-charge-conjugation amplitude analogous to W_2 of electroproduction. Let q_1 and p_1 (q_2 and p_2) be the four-momenta of the incoming (outgoing) current and particle; the amplitude under discussion is

$$T_{\mu\nu} = (2\pi)^3 (4p_1^0 p_2^0)^{1/2} \int e^{iq\cdot x} d^4x \, \langle p_1 | [J_\mu^\alpha(x), J_\nu^\beta(0)] | p_2 \rangle$$

$$= P_\mu P_\nu A(\nu, t, Q^2, \delta) + \cdots, \tag{1}$$

with

$$P = \tfrac{1}{2}(p_1 + p_2), \quad Q = \tfrac{1}{2}(q_1 + q_2), \quad (P^2)^{1/2}\nu = P\cdot Q, \quad t = (p_1 - p_2)^2, \text{ and } \delta = q_2{}^2 - q_1{}^2.$$

The conjectured Bjorken scaling limit for the A amplitude is

$$\lim_{\nu, Q^2 \to\infty;\; Q^2/2\nu = \omega} \nu A(\nu, t, Q^2, \delta) = F(\omega, t). \tag{2}$$

The above limiting relation may be obtained by postulating the appropriate singularity behavior of the product of the currents $J_\mu^\alpha(x)$ and $J_\nu^\beta(0)$ as $x^2 \to 0$. We retain only that part of this product that is kinematically relevant to the A amplitude:

$$\lim_{x^2\to 0}\,[J_\mu^\alpha(x), J_\nu^\beta(0)] = -i\epsilon(x_0)\delta(x^2)\,B_{\mu\nu}(x,0)$$
$$= -i\,\epsilon(x_0)\delta(x^2)\,[\theta^{(2)}_{\mu\nu}|(0) + x^\lambda x^\eta\,\theta^{(4)}_{\mu\nu\lambda\eta}(0) + \cdots], \tag{3}$$

where $B_{\mu\nu}(x, 0)$ is a bilocal operator defined in terms of the local nth-rank tensor operators $\theta^{(n)}_{\mu_1\cdots\mu_n}$.[3,5]

Inserting Eq. (3) into Eq. (1) and performing the limit indicated in Eq. (2), we obtain

$$F(\omega,t) = \pi\int dx_0\, e^{i\omega x_0}\;\epsilon(x_0)(2\pi)^3(4p_1^0p_2^0)^{1/2}\langle p_1|\,B(x,0)\,|p_2\rangle_{|\mathbf{x}|=|x_0|}\,. \tag{4}$$

If the form factor of the nth-rank tensor field $\theta^{(n)}_{\mu_1\cdots\mu_n}$ is defined by

$$(2\pi)^3(4p_1^0p_2^0)^{1/2}\langle p_1|\theta^{(n)}_{\mu_1\cdots\mu_n}(0)|p_2\rangle = P_{\mu_1}\cdots P_{\mu_n}G_n(t)/(p^2)^n, \tag{5}$$

then

$$F(\omega,t) = 2\pi\sum_{n=1}(2n)!\,G_{2n}(t)\left(\frac{1}{\omega}\right)^{2n-1}. \tag{6}$$

A combination of the Regge and Bjorken limits yields[6]

$$F(\omega,t)\underset{1/\omega\to\infty}{\sim}\beta(t)\,(1/\omega)^{\alpha(t)-1}. \tag{7}$$

We shall now construct a Van Hove type[8] model consistent with the above limits and which gives a direct interpretation of the tensor fields $\theta^{(n)}$ appearing in Eq. (3). In the Van Hove model we assume that the amplitude $T_{\mu\nu}$ is obtained from the imaginary part of an infinite sum of t- channel exchanges of increasing mass and spin. Let $\phi^{(J)}_{\alpha\cdots\alpha_J}$ be a field corresponding to a particle of spin J and mass $m(J)$. We assume that $m(J)$ is analytic and monotonic in J. We take as the coupling of this field with the initial and final hadrons of Eq. (1)

$$\beta_1(J)\,\frac{P^{\alpha_1}\cdots P^{\alpha_J}}{(P^2)^{(J-2)/2}}\,\theta_{\alpha_1\cdots\alpha_J}\,, \tag{8a}$$

and with the two currents of Eq.(1)

$$\beta_2(J)\,\frac{Q^{\alpha_3}\cdots Q^{\alpha_J}}{(Q^2)^{J-1}}\,\phi_{\mu\nu\alpha_3\cdots\alpha_J}\,. \tag{8b}$$

The $(Q^2)^J$ in the denominator of Eq. (8b) is the *postulated* behavior necessary to give us the desired Bjorken scaling. In the limit of large ν, Q^2, the amplitude A is found to be

$$A(\nu,t,Q^2,\delta) = \mathrm{Im}\sum_{J\,\mathrm{even}}\beta(J)\left(\frac{2P\cdot Q}{P^2Q^2}\right)^{J-2}\frac{1}{Q^2}\,\frac{1}{t-m^2(J)}, \tag{9}$$

where $\beta^{(J)} = \beta_1^{(J)}\beta_2^{(J)}$ multiplied by numerical factors. Summing the series in Eq.(9) by means of a Watson-Sommerfeld transform we obtain the desired limit of Eq.(7).

We have found that the scaling and Regge limits may be made consistent with each other within a dynamical model, and comparing the coefficients of the $(1/\omega)^J$ terms in Eq.(6) and Eq.(9), we find a direct interpretation of the fields $\theta^{(n)}_{\mu_1\cdots\mu_n}$ as the fields of particles of successively higher spins and masses whose exchanges in the t channel sum up to a Regge-pole contribution.

This analysis does not illuminate the diffractive or Pomeranchuk contribution to the Bjorken-Regge limits. If the vacuum trajectory is like all the other ones and passes through particles, then a similar analysis to the one above may be applied to it. If, on the other hand, diffraction scattering is governed by a flat trajectory, a superposition of cuts, etc., then the above discussion will be valid only for the ordinary exchanges.

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